The Optimal Rounded Cornered Canal Section with Consideration of Free Board

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ABSTRACT

The optimal section for a trapezoidal rounded cornered canal is derived by minimizing the total area of construction for the given discharge. Total area of construction includes flow area and area corresponding to free board above the water surface. The free board required to be provided depends on the discharge of canal. The values of free board for different canal discharges are recommended by USBR as well as by Indian Standards. As per the recommendation of USBR the free board can also be estimated from the depth of flow instead of discharge of canal. The paper develops method for the design of optimal trapezoidal rounded cornered canal section considering the free board based on design discharge of canal as well as the free board estimated from depth of flow. The method suggested is based on numerical trial and error solution.

Key Words: Canal Section, Depth of Flow, Discharge of Canal, Free Board, Trapezoidal Section

I INTRODUCTION

Trapezoidal round cornered section is more efficient than sharp cornered section. The optimal trapezoidal round cornered section is derived by minimizing the total area of construction. The total area of construction includes the flow area and free board area. The free board is considered as discharge dependent recommended by IS 4745-1564 as well as depth dependent recommended by USBR.

Attempts have been made by various researchers to derive the optimal sharp cornered section. The paper deals with consideration of rounding of the corners in trapezoidal section. Lagrangian undetermined multiplier technique is used to derive an optimal trapezoidal round cornered section corresponding to the minimum total area of construction which includes flow area and free board area which can be one of the logical approach of optimal canal design with consideration of free board. The method based on numerical trial and error solution are given in the paper for designing the optimal trapezoidal round cornered canal section which taken into consideration the free board dependent on discharge as well as free board dependent as depth of flow.

II. PARAMETRIC RELATIONSHIP

2.1 Geometrical parameter

The flow area A and wetted perimeter P of a trapezoidal section as shown in fig 1 are expressed or

$A = by + r^2 z_2 + zy^2 + 2 ryz_1 – 2r^2 z_1$ ------------ (1)

And

$P = b + 2y \sqrt{(1+ z^2)} – 2 rz_1 + 2 r z_2$ ------------ (2)

The total area of canal section A_t as given by

$A_t = b (y + f) + r^2 z_2 + z (y + f)^2 + 2 r (y + f) z_1 – 2 r^2 z_1$--------------------- (3)

Where f is free board.

2.2 Flow Equation

For flow computation, Manning's eqn. is used and the section factor Z can be expressed as

$Z = AR^{2/3} = Q_n / \sqrt{So}$ -------------------------- (4)

Where R = hydraulic radius, Q = discharge, n= Manning roughness const, and So = bed gradient.

$AR^{2/3} = Q_n / \sqrt{So}$

$A\,(A/P)^{2/3} = Q_n / \sqrt{So}$
$(A)^{5/3} / (P)^{2/3} = Q_n / \sqrt{So}$

$[(by + r^2 z_2 + zy^2 + 2ryz_1 - 2r^2 z_1)^{5/3}] / [(b + 2y\sqrt{(1 + z^2)} - 2\,rz_1 + 2rz_2)^{2/3}] = Q_n / \sqrt{So}$

$[y^{10/3}\ (b/y + r^2/y^2 * z_2 + z + 2\,r/y * z_1 - 2\,r^2/y^2 * z_1)^{5/3}] / [y^{2/3}\,(b/y + 2\sqrt{(1 + z^2)} - 2\,r/y * z_1 + 2\,r/y * z_2)^{2/3}] = Q_n / \sqrt{So}$

$y^{8/3} = Q_n\,[\,b/y + 2\sqrt{(1 + z^2)} - 2\,r/y\,z_1 + 2\,r/y\,z_2\,]^{2/3} / \sqrt{So}\ [\,b/y + r^2/y^2\,z_2 + z + 2\,r/y\,z_1 - 2\,r^2/y^2\,z_1\,]^{5/3}$

Taking power 3/8

$y = (Q_n)^{3/8}\,[\,b/y + 2\sqrt{(1 + z^2)} - 2\,r/y\,z_1 + 2\,r/y\,z_2\,]^{1/4} / (\sqrt{So})^{3/8}\ [\,b/y + (r/y)^2\,z_2 + z + 2\,r/y\,z_1 - 2\,r^2/y^2\,z_1\,]^{5/8}$ -- (5)

$y = \Phi \times (Q_n / \sqrt{So})^{3/8}$ ------------------------ (6)

Where $\Phi = f\,(b/y,\ z,\ r/y)$

The values of function Φ are calculated for various values of b/y ratio and r/y ratio for the given values of z. (z = 1.0, 1.5, 2.0)

III. OPTIMIZATION APPROACH

The section factor Z is a function of b, y & z & the total area of construction A_t is a function, of b, y, z and assuming side slopes of the canal is constant and determined from stability consideration of the soil. The free board can be considered as a constant for a given discharge or (discharge dependent free board) as a function of a depth of flow (depth dependent free board). The condition for the optimal cross section of the canal can be obtained by using, Lagrangian undetermined multiplier technique as

$dZ / db + \lambda\, dA_t / db = 0$ -------------- (7)

And

$dZ / db + \lambda\, dA_t / dy = 0$ --------------- (8)

Eliminating the unknown multiplier λ from eq (7) & (8) we get

$dZ / db * dA_t / dy = dZ / dy * dA_t / db$

$(dZ / db) / (dZ / dy) = (dA_t / db) / (dA_t / dy)$

$[d\,(AR^{2/3})/db] / [d\,(AR^{2/3})dy] = [dA_t / db] / [dA_t / dy]$ ------------------------------- (9)

IV. Design of Canal Section Provided With Discharge Dependent Free Board.

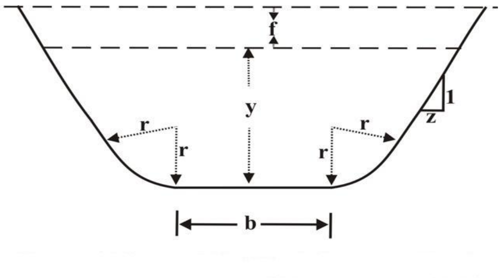

Figure 1 Trapezoidal Round Cornered Section of Canal with Free Board

4.1 Condition for Optimal Canal Section

It is the general practice of providing free board based on the full supply discharge. As discharge remains const in the design free board 'f' is also treated as cost parameter.

As

$A = by + 2\,ry\,z_1 + y^2 z - 2\,r^2 z_1 + r^2 z_2$
$P = b + 2y\sqrt{(1 + z^2)} - 2\,r\,z_1 + 2\,rz_2$

$dA / db = y$ $\quad$ $dP / db = 1$ ------------ (a)

Now considering $AR^{2/3}$ and differentiating w.r.to b

$(d\,AR^{2/3})/db = A\,(2/3)\ R^{-1/3} dR/db + R^{2/3}.dA/db$ ----------------------- (b)

as, $R = A / P$

and

$dR / db = [P\,(dA/db) - A\,(dP/db)] / P^2$

from (a) and (b)

$dR / db = [(b + 2y\sqrt{(1 + z^2)} - 2\,rz_1 + 2\,rz_2)\,y - (\,by + 2ryz_1 + y^2 z - 2\,r^2 z_1 + r^2 z_2)] / P^2$

or

$dR / db = [by + 2\,y^2\sqrt{(1 + z^2)} - 2\,rz_{1y} + 2\,ryz_2 - by - 2ryz_1 - y^2 z + 2\,r^2 z_1 - r^2 z_2] / P^2$

or,

$dR / db = (1/P^2)*(2\,y^2\sqrt{(1 + z^2)} - 4\,r\,yz_1 + 2\,ryz_2 - y^2 z + 2r^2 z_1 - r^2 z_2)$

Multiplying both sides by $AR^{-1/3}$

$AR^{-1/3}\,(dR / db) = (AA^{-1/3}) / (P^2 * P^{-1/3})*(2\,y^2\sqrt{(1 + z^2)} - 4\,ryz_1 + 2\,ryz_2 - y^2 z + 2\,r^2 z_1 - r^2 z_2)$

$= [(A^{2/3}) / P\,(P^{2/3})] * (2\,y^2\sqrt{(1 + z^2)} - 4\,ryz_1 + 2\,ryz_2 - y^2 z + 2\,r^2 z_1 - r^2 z_2)$

$AR^{-1/3}(dR / db) = [(R^{2/3)} / P]*(2y^2\sqrt{(1 + z^2)} - 4 ryz_1 + 2 ryz_2 - y^2z + 2 r^2 z_1 - r^2z_2)$

Put in (b)

$dAR^{2/3} / db = [2/3P] * R^{2/3} (2 y^2\sqrt{(1 + z^2)} -4 ryz_1 + 2 ryz_2 - y^2z + 2r^2z_1 - r^2z_2)+R^{2/3}y$

$=[R^{2/3}/ 3P] * (4y^2\sqrt{(1 + z^2)} - 8ryz_1 + 4ryz_2 - 2y^2z + 4 r^2z_1 - 2 r^2z_2 + 3py)$

$= [R^{2/3}/ 3P] * [4y^2\sqrt{(1 + z^2)} - 8ryz_1+4ryz_2 - 2y^2z+4r^2z_1-2r^2z_2+3(b+2 y \sqrt{(1 + z^2)} -2rz_1+2rz_2) y]$

$= [R^{2/3}/ 3P] * [4y^2\sqrt{(1 + z^2)} - 8ryz_1+4ryz_2 - 2y^2z+4r^2z_1-2r^2z_2+3by+6 y^2 \sqrt{(1 + z^2)} -6ryz_1+6ryz_2]$

$dAR^{2/3} / db = [R^{2/3}/ 3P]* [3by + 10y^2 \sqrt{(1 + z^2)} -14 ryz_1+10 ryz_2 - 2y^2z + 4 r^2z_1 - 2r^2z_2]$-----(c)

Again,

$A = by + 2 ry z_1 + y^2 z - 2r^2z_1 + r^2 z_2$

$P= b+ 2y \sqrt{(1 + z^2)} - 2rz_1+ 2rz_2$

$dA / dy = b + 2 r z_1 + 2 y z$

$dP / dy = 2 \sqrt{(1 + z^2)}$

$R = A / P$

$dR / dy =$ **(**$P.dA/dy - A. dp/dy) / P^2$

$dR/ dy =[1/ P^2]*[b+2y \sqrt{(1 + z^2)} -2 rz_1+2 rz_2) (b+2rz_1+2yz) - (by+2ryz_1+y^2z- 2r^2z_1+r^2z_2)x2 \sqrt{(1 + z^2)}]$

$= [1 / P^2]*[b^2+2brz_1 +2byz+2by \sqrt{(1 + z^2)} + 4ryz_1\sqrt{(1 + z^2)} + 4y^2z \sqrt{(1 + z^2)} - 2rbz_1 - 4r^2z_1^2 - 4 ryzz_1 + 2 rbz_2 + 4r^2z_1z_2 + 4 ryzz_2 - 2by \sqrt{(1 + z^2)} - 4 ryz_1\sqrt{(1 + z^2)} - 2 y^2z \sqrt{(1 + z^2)}+ 4r^2z_1\sqrt{(1 + z^2)} - 2 r^2z_2 \sqrt{(1 + z^2)}]$

$dR/dy = [1 / P^2]* [b^2+2byz +2y^2z \sqrt{(1 + z^2)}- 4r^2z_1^2 - 4 ryz_1z + 2 rbz_2 + 4 r^2 z_1z_2 + 4 ryzz_2 + 4 r^2 z_1 \sqrt{(1 + z^2)} - 2 r^2 z_2 \sqrt{(1 + z^2)}]$

Multiplying both sides by $AR^{-1/3}$

$AR^{-1/3} dR / dy = (AA^{-1/3} / P^{-1/3})* 1 / P^2 [b^2+2byz +2y^2z \sqrt{(1 + z^2)} - 4r^2z_1^2 - 4 ryz_1z +2rbz_2 + 4r^2z_1z_2 + 4ryzz_2 + 4r^2z_1 \sqrt{(1 + z^2)} - 2 r^2 z_2 \sqrt{(1 + z^2)}]$

$AR^{-1/3} dR / dy = (A^{2/3} / P^{2/3})* 1 / P [b^2+2byz +2y^2z \sqrt{(1 + z^2)}- 4r^2z_1^2 - 4 ryz_1z +2rbz_2 + 4r^2z_1z_2 + 4ryzz_2 + 4r^2z_1 \sqrt{(1 + z^2)} - 2 r^2 z_2 \sqrt{(1 + z^2)}]$

$= [R^{2/3} / P]*[b^2+2byz +2y^2z \sqrt{(1 + z^2)} - 4r^2z_1^2 - 4 ryz_1z + 2 rbz_2 + 4 r^2 z_1z_2 + 4 ryzz_2 + 4 r^2 z_1 \sqrt{(1 + z^2)} - 2 r^2 z_2\sqrt{(1 + z^2)}]$

as, $z_1 = \sqrt{(1 + z^2)} - z$

hence $\sqrt{(1 + z^2)} = z_1 + z$

$= [R^{2/3} / P]* [b2+2byz +2y^2z \sqrt{(1 + z^2)} - 4r^2z_1^2 - 4 ryz_1z + 2 rbz_2 + 4 r^2 z_1z_2 + 4 ryzz_2 + 4 r^2 z_1 (z_1 + z) - 2 r^2 z_2 (z1 + z)]$

$AR^{-1/3} dR / dy = [R^{2/3} / P]* [b^2+2byz +2y^2z \sqrt{(1 + z^2)} - 4r^2z_1^2- 4 ryz_1z +2rbz_2 + 4r^2z_1z_2 + 4ryzz_2 + 4r^2z_1^2 + 4r^2zz_1 - 2 r^2z_1z_2 - 2 r^2zz_2]$

$AR^{-1/3} dR / dy = [R^{2/3}/ P]*[b2+2byz +2y^2z \sqrt{(1 + z^2)} - 4ryzz_1 + 4 r^2zz_1 +2rbz_2 + 2 r^2z_1z_2 + 4ryzz_2 - 2 r^2 z z_2]$

$dAR^{2/3} / dy = A 2 /3 * R^{-1/3} dR / dy + R^{2/3} dA/ dy$

$dAR^{2/3} / dy =[2 R^{2/3} / 3P] [b^2+2byz +2y^2z \sqrt{(1 + z^2)} - 4rzz_1y + 4 r^2zz_1 + 2 rbz_2 + 2r^2z_1z_2 + 4 ryzz_2 - 2r^2zz_2] + R^{2/3} (b + 2 rz_1 + 2yz)$

$= [R^{2/3} /3P]* [2b^2+4byz +4y^2z \sqrt{(1+ z^2)} - 8rzz_1y + 8 r^2 zz_1 + 4 rbz_2 + 4 r^2 z_1z_2 + 8 ryzz_2 - 4 r^2 zz_2 + 3P (b + 2 rz_1 + 2yz)$

$=[R^{2/3} / 3P]* [2b^2+4byz +4y^2z \sqrt{(1 + z^2)} - 8rzz_1y + 8 r^2zz_1 + 4rbz_2 + 4r^2z_1z_2 + 8 ryz z_2 - 4 r^2zz_2 +3(b + 2y \sqrt{(1 + z^2)} - 2rz_1 + 2rz_2) (b +2rz_1 + 2yz)]$

$= [R^{2/3} / 3P] *[2b^2+ 4byz +4y^2z \sqrt{(1 + z^2)} - 8rzz_1y + 8 r^2zz_1 + 4 rbz_2 + 4 r^2 z_1z_2 + 8 ryzz_2 - 4 r^2 zz_2 +3b^2 + 6 brz_1 + 6 byz + 6 by \sqrt{(1 + z^2)} + 12 ryz_1\sqrt{(1 + z^2)} + 12 y^2 z \sqrt{(1 + z^2)} - 6 brz_1 - 12 r^2 z_1^2 - 12ryz z_1 + 6 brz_2 + 12 r^2 z_1 z_2 + 12ry z z_2]$

$dAR^{2/3} / dy = [R^{2/3} / 3P]*[5b^2+10byz +16y^2z \sqrt{(1 + z^2)} - 20ryz z_1 + 8r^2zz_1 + 10 rbz_2 +20r^2z_1z_2 + 20 r yz z_2 - 4r^2 z z_2 + 6by \sqrt{(1 + z^2)} + 12ry z_1 \sqrt{(1 + z^2)} - 12 r^2 z_1^2]$ ---------------------------------------(d)

From (c) & (d)

$[dAR^{2/3} / db] / [dAR^{2/3} / dy] = [3by+10y^2 \sqrt{(1 + z^2)} - 14 ry z_1 + 10 ryz_2 - 2y^2 z + 4 r^2 z_1 - 2 r^2 z_2] / [5 b^2 + 10byz + 16 y^2 z \sqrt{(1 + z^2)} - 20 ryzz_1 + 8 r^2 zz_1 + 10rbz_2 + 20r^2z_1z_2 + 20 ryzz_2 - 4 r^2z z_2+ \sqrt{(1 + z^2)} (6by + 12 ryz_1) - 12 r^2 z_1^2]$-------------------(10)

Now considering total area of construction At,

$At = b (y + f) + r^2 z_2 + z (y + f)^2 + 2 r (y + f) z_1 - 2 r^2 z_1$

$dA_t / db = y + f$

$dA_t / dy = b + 2z (y + f) + 2 r z_1$

$[dA_t / db] / [dA_t / dy] = [y + f] / [b + 2 z (y + f) + 2 r z_1]$

$[dA_t / db] / [dA_t / dy] = [1 + f/y] / [b/y + 2 z (1 + f/y) + 2 r/y z_1]$

$[dA_t / db] / [dA_t / dy] = [1 + m] / [b / y + 2 z (1 + m) + 2 r/y z_1]$ -------------------------------- (11)

Equating above expressions (11) and (10) and rearranging, we will get

$(2 +5 m) (b/y)^2 + [6 z - 4 \sqrt{(1 + z^2)} + 8 r/y z_1 - 4 r^2/y^2 z_1 + 2 r^2/y^2z_2 +(4z + 6\sqrt{(1 + z^2)} + 10 r/y z_2) m] b/y + [-4z \sqrt{(1 + z^2)} + 4z^2 + 8 r/y z z_1 + 20 r^2/y^2 z_1z_2 + 12 r/y z_1 \sqrt{(1 + z^2)} - 12 r^2/y^2 z_1^2] (1 +$

m) - 2 r/y z_1 [10 √(1 + z^2) - 14 r/y z_1 + 10 r/y z_2 - 2 z + 4 r^2/y^2 z_1 - 2 r^2/y^2 z_2] = 0

or,

K_1 (b/y $)^2$ + K_2 b/y - { [4z √(1 + z^2) - 4 z^2 - 8 r/y z z_1 - 20 r^2/y^2 z_1 z_2 - 12 r/y z_1 √(1 + z^2) + 12 r^2/y^2 z_1^2] x (1+m) + 2[10 √(1 + z^2) - 14 r/y z_1 + 10 r/y z_2 - 2 z + 4 r^2/y^2 z_1 - 2 r^2/y^2 z_2] r/y z_1 } = 0

K_1 (b/y$)^2$ + K_2 b/y – K_3 = 0

This is quadratic equ in terms of b/y. The value of b/y is given by

b/y = [- K_2+ √ (K_2^2 + 4 K_1 K_3)] / 2 K_1 ------- (13)

Where

K_1 = 2 + 5 m

K_2 = 6 z – 4√(1 + z^2) + 8 r/y z_1 – 4 r^2/y^2 z_1 + 2 r^2/y^2 z_2 + (4z +6 √(1 + z^2) + 10 r/y z_2) m

K_3 = [4z √(1 + z^2) - 4z^2 - 8 r/y z z_1 -20 r^2/y^2 z_1 z_2 - 12 r/y z_1√(1 + z^2) +12 r^2/y^2 z_1^2] (1+m) +2 r/y z_1[10√(1 + z^2)- 14 r/y z_1 +10 r/y z_2 -2 z + 4 r^2/y^2 z_1 -2 r^2/y^2 z_2] -- (14)

Equation (13) and (14) will be used to get the canal section corresponding to the minimum area of construction.

If free board is not considered i.e. f = 0 & m = 0 value of factor

K_1 K_2 & K_3 in equ (14) will be

K_1 = 2.0

K_2 = 6 z – 4 √(1 + z^2) + 8 r/y z_1 – 4 r^2/y^2 z_1 + 2 r^2/y^2 z_2

K_3 = [4 z √(1 + z^2) – 4z^2 - 8 r/y zz_1 –20 r^2/y^2 z_1 z_2 – 12 r/y z_1 √(1 + z^2) + 12 r^2/y^2 z_1^2] + 2 r/y z_1 [10 √(1 + z^2) - 14 r/y z_1 + 10 r/y z_2 –2 z + 4 r^2/y^2 z_1 -2 r^2/y^2 z_2] --(15)

By using equation (13) and (15) the optimal trapezoidal round cornered section can be obtained without considering free board.

4.2 Method Based on Trial and Error

Equ (5) & (13) are used to design the optimal section of a canal. The procedure is iterative in nature. Following steps are adopted to get sol.

1. Assume a trial value of b/y ratio.
2. Calculate value of depth of flow of from equ (5)
3. Compute values of func K1 K2 & K3 by using equ. (14) for given value of z for calculated value of m = f / y and r/y.
4. Calculate value of b/y ratio from equ (13)
5. Compare the calculated value of b/y with assumed value of b/y if they are same the solution is correct. If not, change the value of b/y and repeat steps (i) to IV till the assumed and calculated values of b/y are equal.
6. The section is designed for the corresponding value of b/y ratio from equation (5).

V. Design of Canal Sections Provided With Depth Dependent Free Board

5.1 Condition for Optimal Canal Section

Some times free board f is provided according to the depth of flow and it is generally expressed as

F = k √ y

Where K is const and its value varies bet 0.67 to 0.87 depending on discharge.

Equ (3) can be modified as

A_t = b (y + k √ y) + r^2 z_2 + z (y +k √ y $)^2$ + 2r (y +k √ y) z_1 - 2 r^2 z_1 ---------------------------(16)

d At / db = y + k √ y

d At / dy = b(1 +k/ 2√ y) + 0 +2z(y +k√ y) (1 + k/ 2√ y) + 2 r (1+ k/ 2√ y) z_1

= b (1 + k/ 2√ y) + 2zy + (1+ k/ 2√y + k/√ y + k^2 /2y)+ 2r (1 + k/ 2√ y) z_1

d At / dy = b (1 + k/ 2√ y) + 2 zy (1 + k^2 /2y + 3 k/ 2√ y) + 2 r (1 + k/ 2√ y)z_1

[d At /db] / [d A_t/dy] = [y + k/√ y] / [b (1 + k/ 2√ y) + 2 z y (1 + k^2 /2y + 3 k/ 2√ y) + 2 r(1 + k/ 2√ y) z_1]

[d At /db] / [d A_t/dy] = [1+k/√y] / [b/y (1 + k / 2√y) + 2 z (1 + k^2 /2y + 3 k/ 2√y) + 2 r/y (1 + k/ 2√ y) z_1]

[d At /db] / [d A_t/dy] = [1 + m_1] / [b/y (1 + m_1 / 2) + 2 z (1 + m_1^2 / 2 + 3 m_1 /2) + 2 r/y (1 + m_1 /2) z_1] -------------------------------------- (17)

Where m_1 = k/√ y -------------------- (18)

equating equ (10) & (17) we get

[1 + m_1]/ [b/y (1 + m_1 /2) + 2 z (1 + m_1^2 /2+ 3 m_1 /2) + 2 r/y (1 + m_1 /2) z_1]

= [3 b/y + 10 √(1 + z^2) - 14 r/y z_1 + 10 r/y z_2 - 2z + 4 r^2 z_1 - 2 r^2 z_2] / [5 (b/y$)^2$ + 10 b/y z + 16 z √(1 + z^2) - 20 r/y z z_1 + 8 r^2/y^2 zz_1 +10 r/y b/y z_2 + 20 r^2/y^2 z_1 z_2 + 20 r/y z z_2 - 4 r^2/y^2 z z_2 + 6 b/y √(1 + z^2) + 12 r/y z_1 √(1 + z^2) - 12 r^2 z_1^2

or

(1+m_1) [5(b/y$)^2$ + 10 b/y z + 16 z √(1 + z^2) - 20 r/y z z_1 + 8 r^2/y^2 z z_1 + 10 r/y b/y z_2 + 20 r^2/y^2 z_1 z_2 + 20 r/y z z_2 - 4 r^2/y^2 z z_2 + 6 b/y √(1 + z^2) + 12 r/y z_1√(1 + z^2) - 12 r^2 z_1^2]

= [3 b/y + 10 √(1 + z^2) - 14 r/y z_1 + 10 r/y z_2 - 2z + 4 r^2 /$y^2$$z_1$ -2 $r^{2/}y^2$ z_2]x [b/y (1+m_1 /2) + 2 z (1 + m_1^2/2 + 3 m_1/2) + 2 r/y (1 + m_1 /2) z_1]

By rearranging and solving the above expressions

K_1 (b/y $)^2$ + [6 z - 4 √(1 + z^2) - 8 r/y z_1 4 r^2/y^2 z_1 + 2 r^2/y^2 z_2 + (2 z + √(1 + z^2) + 5 r/y z_2 + 4 r/y z_1 - 2 r^2/y^2 z_1 + r^2/y^2 z_2 - 3 z m_1) m_1] b/y - K_3 = 0

$K_1 (b/y)^2 + K_2\, b/y – K_3 = 0$

This is quadratic equ in terms of b/y, value of b/y can be expressed as

$b/y = [- K_2 + \sqrt{(K_2^2 + 4 K_1 K_3)}] /2 K_1$---------- (19)

Where

$K_1 = 2 + 7 / 2\, m_1$

$K_2 = 6z - 4\sqrt{(1 + z^2)} + 8\, r/y\, z_1 - 4\, r^2/y^2\, z_1 + 2\, r^2/y^2\, z_2 + (2z + \sqrt{(1 + z^2)} +5\, r/y\, z_2 + 4\, r/y\, z_1 -2\, r^2/y^2\, z_1 + r^2/y^2\, z_2 - 3z\, m_1)\, m_1]$

$K_3 = [2z (1 + m_1^2 /2+ 3 m_1/2) + 2\, r/y (1 + m_1/2)\, z_1] [10\sqrt{(1 + z^2)} - 14\, r/y\, z_1 + 10\, r/y\, z_2 - 2z - 4\, r^2/y^2\, z_1 - 2\, r^2/y^2\, z_2] - (1 + m_1) [16\, z \sqrt{(1+ z^2)} - 20\, r/y\, z\, z_1 + 8\, r^2/y^2\, z\, z_1 + 20\, r^2/y^2\, z_1 z_2 + 20\, r/y\, z\, z_2 - 4\, r^2/y^2\, z\, z_2 + 12\, r/y\, z_1 \sqrt{(1 + z^2)} - 12\, r^2/y^2\, z_1^2]$----------- (20)

If free board is not consider ie f = 0 & $m_1 = 0$ the values of factors K_1 , K_2 & K_3 in equ (20) will be

$K_1 = 2.0$

$K_2 = 6\, z - 4 \sqrt{(1 + z^2)} + 8\, r/y\, z_1 - 4\, r^2/y^2\, z_1 + 2\, r^2/y^2\, z_2$

$K_3 = [2\, z + 4\, r/y\, z_1] [10 \sqrt{(1 + z^2)} - 14\, r/y\, z_1 + 10\, r/y\, z_2 - 2\, z - 4\, r^2/y^2\, z_1 - 2\, r^2/y^2\, z_2 - [16\, z \sqrt{(1 + z^2)} - 20\, r/y\, z\, z_1 + 8\, r^2/y^2\, z\, z_1 + 20\, r^2/y^2\, z_1 z_2 + 20\, r/y\, z\, z_2 - 4\, r^2/y^2\, z\, z_2 + 12\, r/y\, z_1 \sqrt{(1 + z^2)} - 12\, r^2/y^2\, z_1^2]$

This is the condition for the best trapezoidal round cornered section corresponding to the minimum wetted perimeter and without considering free board.

5.2 Method Based on Trial and Error

The design of optimal canal section can be obtained by following the same steps as recommended for discharge dependent free board case.

Illustrative Example

1) Discharge dependent free board

Discharge $Q = 100 m^3 /s$

Bed gradient S_0=1 in 6000

Side slope z = 1.5

Manning's roughness constant n = 0.014

Free board f = 0.40

$(Q_n / \sqrt{S_o})^{3/8} = [100*0.014 /\sqrt{(1/6000)}]^{3/8} = 5.7968$

$r^2 / y^2\, z_1 z_2 + 20\, r/y\, z\, z_2 - 4\, r^2 / y^2\, z\, z_2 + 12\, r/y\, z_1 \sqrt{(1+z^2)} - 12\, r2 / y2\, z12]$

$K_3 = 7.9633$

Assume b/y = 0.0296

$\phi = [b/y + 2\sqrt{(1+z^2)} - 2r/yz_1 + 2r/yz_2]^{1/4} / [b/y +(r/y)^2 z_2 + z + 2r/y.z_1 -2(r/y)^2 z_1]^{5/8}$

$\phi = 0.9351$

$y = \phi\ X\ (Q_n / \sqrt{S_o})^{3/8}$

y = 5.42059

m = f/y

m = 0.07379

$k_1 = 2 + 5m$

$k_1 = 2.36895$

$k_2 = 6z-4\sqrt{(1+z^2)} + 8r/y\, z_1 - 4(r/y)^2\, z_1 +2(r/y)^2 z_2+(4z+6\sqrt{(1+z^2)} +10\, r/y\, z_2)\, m$

$k_2 = 5.011859$

$k_3 = [4z\sqrt{(1+z^2)} - 4z^2 - 8\, r/y\, zz_1 - 16(r/y)^2\, z_1 z_2 - 12\, r/y\, z_1\sqrt{(1+z^2)} +12(r/y)^2\, z_1^2] (1+m) + 2\, r/y\, z_1[10\sqrt{(1+z^2)} - 14\, r/y\, z_1 + 10\, r/y\, z_2 – 2z +4 (r/y)^2\, z_1 - 2(r/y)^2 z_2]$

$k_3 = 0.240711$

$b/y = [-K_2+\sqrt{(K_2^2+4K_1K_3)}] / 2K_1$

b/y = 0.04698

b= b/y X y

b = 0.2546

2) Depth Dependant free board: -

$m_1 = K / \sqrt{y}$ | $k = f /\sqrt{y}$ | $m_1 = 0.07379$ | $k = 0.1718$

$K_1 = 2 +7/2\, m_1$

$K_1 = 2+3.5m1$

$K_1 = 2.258265$

$K2 = 6z - 4\sqrt{(1+z^2)} + 8r/y\, z_1 -4(r/y)^2 z_1+2(r/y)^2 z_2+(2z+\sqrt{(1+z^2)} +5\, r/y\, z_2+ 4\, r/y\, z_1 - 2\, (r/y)^2\, z_1 +(r/y)2\, z_2 - 3z\, m_1)\, m_1$

$K_2 = 0.61989$

$K_3 = [2z (1 + m_1^2/2 + 3m_1/ 2) + 2\, r/y (1 + m_1/2)\, z_1] [10\sqrt{(1+z^2)} - 14\, r/y\, z_1 + 10\, r/y\, z_2 - 2\, z - 4\, r^2 / y^2\, z_1 - 2\, r^2 / y^2\, z_2] - (1 +m_1/2) [16\, z \sqrt{(1+z^2)} - 20\, r/y\, z\, z_1 + 8r^2/y^2 zz_1+16$

$b/y = [-K_2+\sqrt{(K_2^2+4K_1K_3)}] / 2K_1$

b/y = 1.7456

b = 9.46218

VI. CONCLUSION

The conditions for the optimal trapezoidal rounded cornered canal section considering free board in the total area of excavation has been developed and the method based on trial and error numerical techniques are suggested. Following conclusions are drawn from the analysis.

1. The discharge dependent free board gives a narrow optimal section as compared to the best section for same discharge i.e. obtained section is with low b/y ratio.

2. The depth dependent free board gives a wider optimal section as compared to the best section i.e. obtained section is with high b/y ratio. Proposed method can be adopted to design the optimal section with consideration of free board.

VII. REFERENCES

[1] V. T. Chow "Open Channel Hydraulics" The McGraw Hill book company New York Nu,1959.

[2] P. N. Modi and S. M. Seth "Design of Most Economical Trapezoidal Section of open Channels" Journal of Irrigation and Power, New Delhi, 1968, pp 271-280.

[3] C. Y. Guo and W. C. Hughes " Optimal Channel Cross Section With Free Board" Journal of Irrigation and Drainage Division ASCE Vol. 110, no.8, 1984, pp 304-314.

[4] A Das "Optimal Channel Cross Section with composite roughness" Journal of Irrigation and Drainage Division ASCE Vol. 126, no.1, 2000, pp 68-71.

[5] G. V. Loganathan "Optimal Design of Parabolic Canals" Journal of Irrigation and Drainage Division, ASCE Vol.117 no.5, 1991, pp 716-735.

[6] P. Monadjemi "General Formulation of Best Hydraulic Channel Section" Journal of Irrigation and Drainage Division ASCE Vol.120, no.1, 1994, pp 27-35.

[7] V. L. Streeter and E. B. Wylie "Fluid Mechanics" Mc Graw Hills Inc, New York Ny, 1979.

[8] S. L. Atmapoojya and R. N. Ingle "The Optimal Canal Section With Consideration of Free Board" IE (I) Journal- CV, Vol.83, Feb-2003.